Possibility of Inhomogeneous Coupling Leading to Decoherence in an Electromagnetically-Induced-Transparency Quantum-Memory Process

Xiong-Jun Liu[a,b*], Zheng-Xin Liu[b,c], Xin Liu[b,c] and Mo-Lin Ge[b,c]
a. Department of Physics, National University of Singapore, 2 Science Drive 3, Singapore 117542
b.Theoretical Physics Division, Nankai Institute of Mathematics,
Nankai University, Tianjin 300071, P.R.China
c. Liuhui Center for Applied Mathematics,
Nankai University and Tianjin University, Tianjin 300071, P.R.China

The effect of inhomogeneous coupling between three-level atoms and external light fields is studied in the electromagnetically induced transparency (EIT) quantum memory techqnique. By introducing a subensemble-atomic system to deal with present inhomogeneous coupling case, we find there is a non-symmetric dark-state subspace (DSS) that allows the EIT quantum memory technique to function perfectly. This shows that such memory scheme can work ideally even if the atomic state is very far from being a symmetric one.

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I. INTRODUCTION

Recently, the novel mechanism of electromagnetically induced transparency (EIT) [1] and its many important applications have attracted much attention in both experimental and theoretical aspects [2, 3, 4, 5]. In particular, based on the "dark-state polaritons" (DSPs) theory [6], the quantum memory via EIT technique is actively being explored by transferring the quantum states of photon wave-packet to metastable collective atomic-coherence (collective quasi spin states) in a loss-free and reversible manner [7]. Quantum information processing based on the interaction between light fields and large number of atoms indicates a large dynamical symmetry [8]. For the three-level EIT quantum memory technique, a semidirect product group under the condition of large atom number and low collective excitation limit [6] is discovered by Sun *etal*. [9], and the validity of adiabatic condition for the evolution of DSPs is also confirmed. Subsequently, a series of works are involved in the hidden dynamical symmetry and the applications to quantum information processing, such as the generation of quantum entanglement between atoms (or lights) with multi-level atomic system (or multi-atomic-ensemble system) [10, 11, 12], etc.

However, to realize such a large dynamicl symmetry in the field-atoms interaction model, the coupling between the external fields and atomic ensemble is usually assumed to be homogeneous in DSP theory, i.e. the effect of inhomogeneity for atoms with different spatial positions is ignored. Very recently, Sun *etal*. [13] investigated the decoherence for a superposition of symmetric internal states of a two-level atomic gas due to the inhomogeneous coupling with external light fields. Within their model, for an ensemble composed of N atoms, they find that the apparent decoherence or dissipation rate for superpositions of collective spin states scales as $\sqrt{N}$. On the other hand, the applications to

* Electronic address: phylx@nus.edu.sg

magnetometery with inhomogeneous coupling between the two-level atomic ensembles and light field was studied by A. Kuzmich et al [14]. Also, the effect of inhomogeneous coupling in spin squeezing and precision measurement with light was studied by L. B. Madsen and K. Mølmer [15]. As a result, the effect of inhomogeneous coupling in EIT quantum memory process naturally becomes an important issue, especially about the validity in this memory process with collective spin states [13], which we proceed to study in the following.

In this paper, we show the model for three-level EIT quantum memory with inhomogeneous coupling can be equivalently treated as a many-atomic-ensemble system (sub-ensemble atomic system), and then a non-symmetric dark-state subspace (DSS) of present system can be exactly obtained. Although the leakage coefficient will be larger than zero after the quantized probe light is stored, the quantum states of the probe light can be fully recovered when it is released by turning on the control field again.

II. MODEL

We consider a quasi one-dimensional model for an ensemble of N identical atoms with the three-level Λ type structure. The j-th atom interacts with an input quantized field with coupling constant g_j, and the classical control filed with time-space-dependent Rabi-frequency $\Omega(z_j, t)$. The interaction hamiltonian is

$$\hat{H} = -\sum_{j}^{N} g_j \Big(\hat{\sigma}_{ab}^{j} \, \hat{E}^{(+)}(z_j, t) + h.a \Big) - \hbar \sum_{j}^{N} \Big(\hat{\sigma}_{ac}^{j} \, \Omega(z_j, t) \, \mathrm{e}^{i(k_c z_j - \nu_c t)} + h.a \Big) \tag{1}$$

where g_j and $\Omega(z_j)$ are the inhomogeneous coupling constants between the j-th atom and probe and control fields. Although here we consider the inhomogeneous coupling case, generally the coupling constants should be slowly varying with the adjacent atoms. For this we make an approximation that in a small length Δz_σ around the point z_σ there are $N_\sigma \gg 1$ atoms which interact with the quantized field with a homogeneous coupling constant g_σ, and the classical control filed with time-dependent Rabi-frequency $\Omega(z_\sigma, t)$. Therefore, the present model with inhomogeneous coupling can be equivalently described as the many-atomic-ensemble case (see the ref. [12] section II, part B), i.e. all the atoms in the length Δz_σ (with atom number N_σ) can be approximately described as a single atomic ensemble with the same coupling constant g_σ and Rabi-frequency $\Omega_\sigma(t) = \Omega(z_\sigma, t)$ (while the couplings are different between different ensembles)

$$\sum_{z_j \in N_\sigma} g_j \hat{\sigma}_{\mu\nu}(z_j) \rightarrow g_\sigma \sum_{z_j \in N_\sigma} \hat{\sigma}_{\mu\nu}(z_j), \tag{2}$$

$$\sum_{z_j \in N_\sigma} \Omega(z_j, t) \hat{\sigma}_{\mu\nu}(z_j) \rightarrow \Omega_\sigma(t) \sum_{z_j \in N_\sigma} \hat{\sigma}_{\mu\nu}(z_j). \tag{3}$$

This assumption is reasonable. Because, for example, we can consider the case that the atoms in a very small volume ΔV around z_σ point (the number is still larger larger than 1, see e.g. [6, 7]) have the nearly equal coupling constants, so that the coupling in the small volume can be treated as homogeneous (we may call this the quasi-inhomogeneous case). Then, considering all transitions

at resonance and for the single-mode probe field case, i.e. $\hat{E}^{(+)}(z_j,t) = \hat{a}e^{i(k_p z_j - \nu_p t)}$, the interaction Hamiltonian can be rewritten as:

$$\hat{H} = \sum_{\sigma=1}^{m} g_\sigma \sqrt{N_\sigma}\hat{a}\hat{A}_\sigma^\dagger + \sum_{\sigma=1}^{m} \Omega_\sigma(t)\hat{T}_\sigma^{+} + h.c., \tag{4}$$

where $N_1 + N_2 + ... + N_m = N$ and the subscript $\sigma = 1, 2, 3, ..., m$ denotes the corresponding atomic ensemble and the collective atomic excitation operators:

$$\hat{A}_\sigma = \frac{1}{\sqrt{N_\sigma}} \sum_{z_j \in N_\sigma} e^{-i(k_{ba}z_j - \omega_{ba}t)}\hat{\sigma}_{ba}^{j(\sigma)}, \quad \hat{C}_\sigma = \frac{1}{\sqrt{N_\sigma}} \sum_{z_j \in N_\sigma} e^{-i(k_{bc}z_j - \omega_{bc}t)}\hat{\sigma}_{bc}^{j(\sigma)} \tag{5}$$

with $\hat{\sigma}_{\mu\nu}^{i} = |\mu\rangle_{ii}\langle\nu|(\mu,\nu = a,b,c)$ being the flip operators of the i-th atom between states $|\mu\rangle$ and $|\nu\rangle$, $\mathbf{k_{ba}}$ and $\mathbf{k_{ca}}$ are, respectively, equal the wave vectors of the quantum and classical light fields, $\mathbf{k_{bc}} = \mathbf{k_{ba}} - \mathbf{k_{ca}}$ and

$$\hat{T}_\sigma^{-} = (\hat{T}_\sigma^{+})^\dagger = \sum_{z_j \in N_\sigma} e^{-i(k_{ca}z_j - \omega_{ca}t)}\hat{\sigma}_{ca}^{j(\sigma)}. \tag{6}$$

Denoting by $|b^{(\sigma)}\rangle = |b_1^{(\sigma)}, b_2^{(\sigma)}, ..., b_{N_\sigma}^{(\sigma)}\rangle(\sigma = 1,2,...,m)$ the collective ground state of the σ-th atomic ensemble with all atoms staying in the same single particle ground state $|b\rangle$, we can easily give other quasi-spin wave states by the operators defined in formula (5): $|a_{(\sigma)}^{n}\rangle = [n!]^{-1/2}(\hat{A}_\sigma^\dagger)^n|b^{(\sigma)}\rangle$ and $|c_{(\sigma)}^{n}\rangle = [n!]^{-1/2}(\hat{C}_\sigma^\dagger)^n|b^{(\sigma)}\rangle$. Similarly, in large N_σ limit and low excitation condition, it follows that $[\hat{A}_{(i)}, \hat{A}_{(j)}^\dagger] = \delta_{ij}, [\hat{C}_{(i)}, \hat{C}_{(j)}^\dagger] = \delta_{ij}$ and all the other commutators are zero, which shows the mutual independence between these bosonic operators $\hat{A}_i$ and $\hat{C}_i$. On the other hand, one can easily find the commutation relations: $[\hat{T}_i^{+}, \hat{T}_j^{-}] = \delta_{ij}\hat{T}_j^{z}$ and $[\hat{T}_i^{z}, \hat{T}_j^{\pm}] = \pm\delta_{ij}\hat{T}_j^{\pm}$, where

$$\hat{T}_\sigma^{z} = \sum_{z_j \in N_\sigma} (\hat{\sigma}_{aa}^{j(\sigma)} - \hat{\sigma}_{cc}^{j(\sigma)})/2, \ (\sigma = 1,2,...,m) \tag{7}$$

are the traceless operators.

Following the results obtained in refs. [12], the non-symmetric DSP operator of present system can be defined as

$$\hat{d} = \cos\theta\hat{a} - \sin\theta\prod_{j=1}^{m-1}\cos\phi_j\hat{C}_1 - \sin\theta\sum_{k=2}^{m}\sin\phi_{k-1}\prod_{j=k}^{m-1}\cos\phi_j\hat{C}_k, \tag{8}$$

where the mixing angles θ and ϕ_j are defined through

$$\tan\theta = \frac{\left[\sum_{j=1}^{m}\left(g_j^2 N_j \prod_{k=1,k\neq j}^{m}\Omega_k^2\right)\right]^{1/2}}{\Omega_1\Omega_2...\Omega_m} \tag{9}$$

and

$$\tan\phi_j = \frac{g_{j+1}\sqrt{N_{j+1}}\prod_{k=1}^{j}\Omega_k}{\left[\sum_{k=1}^{j}\left(g_k^2 N_k \prod_{s=1,s\neq k}^{j+1}\Omega_s^2\right]^{1/2}\right.} \tag{10}$$

From the equation (10) one finds $\tan\phi_1 = g_2\sqrt{N_2}\Omega_1/g_1\sqrt{N_1}\Omega_2$, $\tan\phi_2 = g_3\sqrt{N_3}\Omega_1\Omega_2/\sqrt{g_1^2N_1\Omega_2^2\Omega_3^2 + g_2^2N_2\Omega_1^2\Omega_3^2}...$, etc. Also, by a straightforward calculation one can verify that $[\hat{d},\hat{d}^\dagger] = 1$ and $[\hat{H},\hat{d}\ ] = 0$, hence the general atomic dark states can be obtained through $|D_n\rangle = [n!]^{-1/2}(\hat{d}^\dagger)^n|b^{(1)}, b^{(2)}, ..., b^{(m)}\rangle_{atom} \otimes |0\rangle_{photon}$, where the collective ground state $|b^{(\sigma)}\rangle = |b_1, b_2, ..., b_{N_\sigma}\rangle$ and $|0\rangle_{photon}$ denotes the electromagnetic vacuum of the quantized probe field. With the exact dark state obtained in present inhomogeneous EIT model, we can readily study the effect of inhomogeneous coupling between atoms and external fields.

III. THE EFFECT OF THE INHOMOGENEOUS COUPLING

To verify the effect on the quantum memory processing due to the inhomogeneous coupling constant, we rewrite the Hamiltonian of the formular (4) as $H = H_0 + H_1$ with

$$\hat{H}_0 = g_0\sqrt{N}\hat{a}\hat{A}^\dagger + \sum_{\sigma=1}^{m}\Omega_0(t)\hat{T}_\sigma^+ + h.c., \tag{11}$$

$$\hat{H}_1 = \sum_{\sigma=1}^{m}\delta_\sigma\sqrt{N_\sigma}\hat{a}\hat{A}_\sigma^\dagger + \sum_{\sigma=1}^{m}\lambda_\sigma\hat{T}_\sigma^+ + h.c., \tag{12}$$

where $\hat{A} = \sum_{\sigma=1}^{m}\sqrt{N_\sigma}\hat{A}_\sigma/\sqrt{N}$, $\hat{C} = \sum_{\sigma=1}^{m}\sqrt{N_\sigma}\hat{C}_\sigma/\sqrt{N}$, $g_\sigma = g_0 + \delta_\sigma$ and $\Omega_j = \Omega_0 + \lambda_\sigma$. δ_σ and λ_σ denote the inhomogeneous part of the coupling of the probe and control fields, respectively. For the usual three-level EIT technique, the coupling is assumed homogeneous and then $H_1 = 0$. Noting that

$$\hat{U}_0(t) = \exp(-i\hat{H}_0 t), \quad \hat{U}(t) = \exp(-i\hat{H}t) \tag{13}$$

are the corresponding time evolution operators. For some initial state $|\psi(0)\rangle$, the operator $\hat{U}_0$ leads to the evolution $|\psi(t)\rangle_0 = \hat{U}_0(t)|\psi(0)\rangle$, while the actual evolution reads $|\psi(t)\rangle = \hat{U}(t)|\psi(0)\rangle$.

According to the treatment of ref. [13], the leakage of the quantum information is defined as

$$\xi = 1 - |\langle\psi(t)|\psi_0(t)\rangle|^2. \tag{14}$$

$\xi \to 0$ means no leakage, while $\xi \to 1$ indicates a complete loss of the system coherence and population. For a two-level atomic ensemble system [13], the leakage or decoherence of the collective spin states is dependent on the inhomogeneous coupling and number of the atoms. Then there is no advantage of using collective spin states for quantum information processing in that system. However, as the discussions in the following, we will show this is different from the quantum memory processing via EIT technique which is concerned with a three-level atomic ensemble system.

Considering the case of the quantum memory for photons with present EIT technique, the initial total state reads (meanwhile $\theta = 0$ or the external control field is very strong):

$$|\Psi(0)\rangle = \sum_n C_n|n\rangle_{photon} \otimes |b^{(1)}, b^{(2)}, ..., b^{(m)}\rangle_{atom}, \tag{15}$$

where $|\phi_0\rangle = \sum_n C_n|n\rangle$ is the initial quantum state of the quantized probe field and the collective ground states $|b^{(\sigma)}\rangle = |b_1, b_2, ..., b_{N_\sigma}\rangle$. To facilitate further discussion, we decompose the quantum state of photons in terms of the basis of coherent states: $|\phi_0 = \sum_n C_n|n\rangle = \sum_j C'_j|\alpha^{(j)}\rangle$, thus we have

$$|\Psi(0)\rangle = \sum_j C'_j|\alpha^{(j)}\rangle_{photon} \otimes |b^{(1)}, b^{(2)}, ..., b^{(m)}\rangle_{atom}, \tag{16}$$

where $|\alpha^{(j)}\rangle = \sum_n P_n(\alpha^{(j)})|n\rangle$ with $P_n(\alpha^{(j)}) = \frac{(\alpha^{(j)})^n}{\sqrt{n!}}e^{-|\alpha^{(j)}|^2/2}$ is the probability of distribution function. After the quantum memory process that the mixing angle θ is adiabatically rotated from 0 to $\pi/2$, the operator $\hat{U}_0(t)$ leads to the resultant state

$$|\Psi(t)\rangle_0 = |0\rangle_{photon} \otimes \sum_j C'_j|\alpha_1^{(j)}, \alpha_2^{(j)}, ..., \alpha_m^{(j)}\rangle_{coherence}, \tag{17}$$

where $\alpha_k^{(j)}/\alpha_l^{(j)} = \sqrt{N_k}/\sqrt{N_l}$ and $|\alpha^{(j)}|^2 = |\alpha_1^{(j)}|^2 + |\alpha_2^{(j)}|^2 + ... + |\alpha_m^{(j)}|^2$. One can straightly verify that on Fock-state basis of the total atomic system, the above result can be rewritten as $|\Psi(t)\rangle_0 = |0\rangle_{photon} \otimes \sum_j C'_j |\alpha^{(j)}\rangle_{coherence} = |0\rangle_{photon} \otimes \sum_n C_n |n\rangle_{coherence}$, which means that the quantum states of the atom coherence are the same with that of input probe light. However, the actual evolution is governed by $|\psi(t)\rangle = \hat{U}(t)|\psi(0)\rangle$. Thus the actual final state is

$$|\Psi(t)\rangle = |0\rangle_{photon} \otimes \sum_j C'_j |\bar{\alpha}_1^{(j)}, \bar{\alpha}_2^{(j)}, ..., \bar{\alpha}_m^{(j)}\rangle_{coherence}, \tag{18}$$

where $\bar{\alpha}_k^{(j)}/\bar{\alpha}_l^{(j)} = (g_k\sqrt{N_k}/g_l\sqrt{N_l}) \lim_{\Omega_k,\Omega_l \to 0} \Omega_k(t)/\Omega_l(t)$ and $|\alpha^{(j)}|^2 = |\bar{\alpha}_1^{(j)}|^2 + |\bar{\alpha}_2^{(j)}|^2 + ... + |\bar{\alpha}_m^{(j)}|^2$.

It is easy to see that if $g_1 = g_2 = ... = g_m$ and $\Omega_1 = \Omega_2 = ... = \Omega_m$, one has $\bar{\alpha}_k^{(j)} = \alpha_k^{(j)}$ and $|\langle\Psi(t)|\Psi_0(t)\rangle|^2 = 1$. But for the inhomogeneous coupling case, one can verify that generally $|\langle\Psi(t)|\Psi_0(t)\rangle|^2 < 1$, i.e. for present case

$$\xi = 1 - |\langle\Psi(t)|\Psi_0(t)\rangle|^2 > 0. \tag{19}$$

One can give an intuitive understanding of above results. Since our model should be treated as multi-ensemble atomic system, the quantum information is stored in many atomic ensembles when the control field is turned off. Because the couplings of different atomic ensembles are different, the quantum information of photons cannot be stored "homogeneously" in different atomic ensembles, i.e. it is divided into many inhomogeneous parts in the atoms. In other words, a non-symmetric atomic state is prepared after the photons are stored. This is why there is infidelity during the storage process. However, the result $\xi > 0$ does not mean that there is leakage of the coherence in the quantum memory process. In fact, when the control field is adiabatically turned on again, i.e. the mixing angle θ is rotated adiabatically from $\pi/2$ to 0, with the dark-state evolution we find the quantum states of the photons can be recovered from the atom coherence

$$\begin{gathered} |\Psi(t)\rangle \to |\Psi(0)\rangle : \\ |0\rangle_{photon} \otimes \sum_j C'_j |\bar{\alpha}_1^{(j)}, \bar{\alpha}_2^{(j)}, ..., \bar{\alpha}_m^{(j)}\rangle_{coherence} \longrightarrow \\ \longrightarrow \sum_n C_n |n\rangle_{photon} \otimes |b^{(1)}, b^{(2)}, ..., b^{(m)}\rangle_{atom}, \end{gathered} \tag{20}$$

The above derivation clearly shows that the quantum memory scheme is still reversible in the inhomogeneous case. Even the storage states of the atom coherence is different form the that of the input quantized probe light due to the inhomogeneous coupling, the quantum states of the photons can be recovered in the released process when the mixing angle θ is rotated back to 0 by turning on the control field. This means that there really is advantage of using collective spin states for quantum memory and quantum information processing with EIT technique.

We make a few remarks on above results: from the eq. (4) one can see that the inhomogeneous coupling leads to asymmetry in the interaction Hamiltonian, i.e. $\hat{H}$ is not symmetric with respect to permutation $\hat{A}_j \leftrightarrow \hat{A}_k$ (or $\hat{T}_j \leftrightarrow \hat{T}_k$) of any two collective atomic operators, thus the atomic state provided by this Hamiltonian is also nonsymmetric. Since the initial state of the probe photons is generally symmetric, the asymmetry of $\hat{H}$ will lead to "leakage" of the quantum information when the photons are stored into the atomic coherence. This is in agreement with the two-level case [13]. However, as we have shown above, the dark state for our EIT model exists even in the inhomogeneous

coupling case, thus the total state can evolve back into the initial one and the quantum states of the photons can fully be recovered. Because there is no dark state in two-level system, this phenomenon does not occur in Sun *etal* case and the leakage of quantum information is hard to cancel out. The perfect character of EIT quantum memory scheme with inhomogeneous coupling also accounts for the existence of decoherence-free subspace (DFS) [8, 16, 17, 18, 19] (i.e. here it is the DSS) in this case. Finally, since the nonsymmetric entangled atomic states may be observed with no decoherence induced by the inhomogeneous coupling [14], the issue to measure the quantum states of the atom coherence with nonsymmetric observables after the photons are stored for current EIT model will be interesting and deserve further study.

Before conclusion we should emphasize that here we consider the quasi-inhomogeneous coupling case, which of course, is reasonable for many practical cases. However, for the most general case that every atom has a different coupling constant from others, there may really be docoherence in the quantum memory process with EIT. This is also an interesting issue that will be studied in our future publications.

IV. CONCLUSIONS

We have discussed in detail the effect of inhomogeneous coupling in a three-level EIT quantum memory process. The current model is shown to be equivalent to a many-atomic-ensemble (sub-ensembles) one, for which a DSS can exist even if the atomic state is far from being a symmetric one. Although the inhomogeneity can lead to a non-zero leakage during the storage process for the quantum information of probe light, the quantum states can be fully recovered in the released process due to the existence of present non-symmetric DSS. This means that the EIT quantum memory technique functions perfectly even for the inhomogeneous coupling case. Furthermore, the model with sub-ensembles of atoms having the same coupling to the field is powerful and in principle able to deal with propagational and light scattering effects that are often ignored (see, e.g. [15]). Finally, based on our sub-ensemble model we shall be able to propose a general algebraic method to study all kinds of cases in the EIT technique, which will be useful for probing the applications to quantum memory and quantum information processing.

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